Exercise 25

Find the derivative of the function.

$$g(u) = \left(\frac{u^3 - 1}{u^3 + 1}\right)^8$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$g'(u) = \frac{dg}{du} = \frac{d}{du}\left[\left(\frac{u^3 - 1}{u^3 + 1}\right)^8\right]$$

$$= 8\left(\frac{u^3 - 1}{u^3 + 1}\right)^7 \cdot \frac{d}{du}\left(\frac{u^3 - 1}{u^3 + 1}\right)$$

$$= 8\left(\frac{u^3 - 1}{u^3 + 1}\right)^7 \cdot \frac{\left[\frac{d}{du}(u^3 - 1)\right](u^3 + 1) - \left[\frac{d}{du}(u^3 + 1)\right](u^3 - 1)}{(u^3 + 1)^2}$$

$$= 8\left(\frac{u^3 - 1}{u^3 + 1}\right)^7 \cdot \frac{(3u^2)(u^3 + 1) - (3u^2)(u^3 - 1)}{(u^3 + 1)^2}$$

$$= 8\left(\frac{u^3 - 1}{u^3 + 1}\right)^7 \cdot \frac{6u^2}{(u^3 + 1)^2}$$

$$= \frac{48u^2(u^3 - 1)^7}{(u^3 + 1)^9}$$